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HALF YEARLY EXAMINATION–2023-24

CLASS–XI

SUBJECT–MATHEMATICS (041)

Time : 3 Hours M.Marks : 80

General Instructions:

1. *This Question paper contains five sections A, B, C,D and E. Each Section is compulsory. However there are internal choices in some questions.*
2. *Section-A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.*
3. *Section-B has 5 Very short Answer type (VSA) questions of 2 marks each.*
4. *Section-C has 6 Short Answer Type questions (SA) of 3 marks each.*
5. *Section-D has 4 Long Answer Type questions (LA) of 5 marks each.*
6. *Section-E has 3 source based or case based questions of 4 marks each with sub-parts.*

SECTION–A

(Multiple Choice Questions)

Each question carries 1 mark.

1. Which of the following set is Empty ?

   A. $\{x : x \text{ is an integer, } x^2 \leq 9\}$

   B. $\{x : x \text{ is rational and } x^2 - 2 = 0\}$

   C. The collection of all Hindi Dramas written by Shakespeare

   D. $\{x : x \in Z, |x| \leq 12\}$

2. Which of the following interval represent the Set R – (–1, 1).

   (a) (–1, 1) (b) [–1, 1]

   (c) $(-\infty, -1) \cup (1, \infty)$ (d) $(-\infty, -1] \cup [1, \infty)$

3. Which of the following is Singleton set ?

A. $\{x : |x| < 4, \quad x \in N\}$

B. $\{x : 2 < x < 4, \quad x \in N\}$

C. Set of all Prime numbers between 15 and 21.

D. $\{x : 5 < x < 6, \quad x \in R\}$

4. If R is a Relation from a Set A to a Set B then

A. $R = A \cup B$

B. $R = A \cap B$

C. $R \subset A \times B$

D. $R \subseteq A \times B$

5. Consider the Relation $R = \{(a, b) : a, b \in Z \text{ and } b = 2a - 4\}$
If $(a, -2)$ & $(4, b) \in R$ then value of a and b is

A. $a = 4 \quad b = 1$

B. $a = 1 \quad b = 4$

C. $a = -1 \quad b = -4$

D. $a = -4 \quad b = -1$

6. Evaluate $\tan 1° \tan 2° \tan 3° \ldots \tan 89°$

(a) 1

(b) 0

(c) ½

(d) Not defined

7. Which of the following is not correct.

(a) $\sin \theta = \frac{-1}{5}$

(b) $\cos \theta = 1$

(c) $\sec \theta = \frac{1}{2}$

(d) $\tan \theta = 20$

8. If $\tan \theta = \frac{1}{2}$ and $\tan \phi = \frac{1}{3}$ then the value of $\theta + \phi$ is

(a) $\frac{\pi}{6}$

(b) π

(c) 0

(d) $\frac{\pi}{4}$

9. The value of $\sin(45° + \theta) - \cos(45° - \theta)$ is

(a) $\sqrt{2}\cos\theta$

(b) 1

(c) 0

(d) $\sqrt{2}\sin\theta$

10. If ${}^{n}C_{12} = {}^{n}C_{8}$ then n is equal to

(a) 20

(b) 12

(c) 6

(d) 30

11. The total number of 9 digit numbers which have all different digits is

(a) 10!

(b) 9!

(c) $9 \times 9!$

(d) $10 \times 10!$

12. If z is a complex number then which of the following is true.

(a) $\overline{z} = z$

(b) $z = |z|^2$

(c) $z = \overline{z} \Leftrightarrow z$ is purely Real

(d) $z = \overline{z} \Leftrightarrow z$ is purely Imaginary

13. $\left(\sqrt{-2}\right)\left(\sqrt{-3}\right)$ is equal to

(a) $\sqrt{6}$

(b) $-\sqrt{6}$

(c) $\sqrt{6}\,i$

(d) None of these

14. The multiplicative inverse of $2 - 3i$ is

(a) $2 + 3i$

(b) $\frac{2}{13} + \frac{3}{13}i$

(c) $\frac{2}{13} - \frac{3}{13}i$

(d) $3 + 2i$

15. $(a + b)^5$ is equal to

(a) $a^5 + b^5$

(b) ${}^5C_0\ a^5 + {}^5C_1\ a^4b + {}^5C_2a^3b^2 + {}^5C_3a^2b^3 + {}^5C_4b^4$

(c) ${}^5C_0 + {}^5C_1\ a^4b + {}^5C_2a^3b^2 + {}^5C_3a^2b^3 + {}^5C_4ab^4 + {}^5C_5$

(d) $a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5$.

16. Number of terms in the explansion of $[(2x + y^3)^2]^3$ is

(a) 3 (b) 7

(c) 6 (d) 4

17. Solution set for Inequality $\frac{1}{x-2} < 0$ is

(a) $(2, \infty)$ (c) ϕ

(b) $(0, 2)$ (d) $(-\infty, 2)$

18. If $-4x > 20$ and $x \in Z^+$ then x belongs to

(a) $\{-6, -7, -8 \ldots.\}$ (b) ϕ

(c) $\{-4, -3, -2, -1\}$ (d) $\{1, 2, 3, 4 \ldots\}$

ASSERTION-REASON BASED QUESTIONS

In the following questions a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

(a) Both A and R are true, and R is the correct explaination of A.

(b) Both A and R are true and R is not the correct explanation of A.

(c) A is true but R is false.

(d) A is false but R is true.

19. Assertion (A) : If $\tan x = -\sqrt{3}$ and x lies in quadrant IV then $x = \frac{5\pi}{3}$

Reason (R) : $\tan \frac{\pi}{3} = \sqrt{3}$ and $\tan(\pi + x) = +\tan x$.

20. Assertion (A) : $f = \{(ab, a + b) : a, b \in Z\}$ is a function.
Reason (R) : $f : A \to B$ is a function iff each element of A has a unique image in B.

Section-B
(Very Short Answer Type Questions)

Each question carries two marks.

21. Find the Range of $f(x) = \sqrt{x^2 + 4}$

22. Using Binomial theorem, Indicate which number is larger $(1.2)^{4000}$ or 800.

23. A circular wire of radius 3 cm is cut and bent so as to lie along the circumference of a hoop whose radius is 48 cm. Find the angle in degrees which is subtended at the centre of hoop.

24. If $\sin x = \frac{3}{5}$, $\cos y = \frac{-12}{13}$ where x and y both lie in second quadrant find the value of $\sin (x + y)$

25. Evaluate : $$\frac{\sin(\pi + \theta)\cos\left(\frac{\pi}{2} + \theta\right)\tan\left(\frac{3\pi}{2} - \theta\right)\cot(2\pi - \theta)}{\sin(2\pi - \theta)\cos(2\pi + \theta)\operatorname{cosec}(-\theta)\sin\left(\frac{3\pi}{2} + \theta\right)}$$

OR

Prove that : $$\frac{\tan\theta + \sec\theta - 1}{\tan\theta - \sec\theta + 1} = \frac{1 + \sin\theta}{\cos\theta}$$

Section-C
(Short Answer Type Questions)

Each question carries three marks.

26. A bag contains six white marbles and five red marbles. Find the number of ways in which four marbles can be drawn from the bag if
    (a) they can be of any colour
    (b) two must be white and two red
    (c) They must all be of the same colour.

27. Find $(a + b)^4 + (a - b)^4$. Hence evaluate $\left(\sqrt{2} + \sqrt{3}\right)^4 + \left(\sqrt{2} - \sqrt{3}\right)^4$.

28. A solution is to be kept betwen 40°C and 45°C. What is the range of temperature in degree fehrenheit, if the conversion formula is $F = \frac{9}{5}C + 32$.

29. Prove that $\sin 10° \sin 30° \sin 50° \sin 70° = \frac{1}{16}$.

OR

Prove that $\frac{\sec 8\theta - 1}{\sec 4\theta - 1} = \frac{\tan 8\theta}{\tan 2\theta}$.

30. If $\tan x = \frac{-4}{3}$ and $\frac{\pi}{2} < x < \pi$, find the values of

(i) $\sin \frac{x}{2}$ (ii) $\cos \frac{x}{2}$

31. If $z_1 = 3 - 2i$, $z_2 = 1 - i$ find $\left|\frac{z_1 + z_2 + i}{z_1 - z_2 - i}\right|$

Section-D

(Long Answer Type Questions)

Each question carries five marks.

32. Prove that $\frac{\sin 5x - 2\sin 3x + \sin x}{\cos 5x - \cos x} = \text{cosec } 2x - \cot 2x$

OR

Prove that $\frac{\sin 8x \cos x - \sin 6x \cos 3x}{\cos 2x \cos x - \sin 4x \sin 3x} = \tan 2x$

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33. Find the number of arrangements of the letters of the word 'INDEPENDENCE'. In how many of these arrangements

(i) do the vowels never occur together ?

(b) do the words begin with I and end in P ?

34. Find all non-zero complex numbers z satisfying $\bar{z} = iz^2$.

35. Show that $2^{4n+4} - 15n - 16$ is divisible by 225 when $n \in N$.

Section-E

(This Section comprises of three case-study / passage based questions of 4 marks each with sub-parts.

36. Case Study 1 : Read the following passage and answer the questions below:

While solving a typical equation, a person finds that one of the root of the equation is a complex number $z = \frac{1+2i}{1-3i}$, help him to find

(i) The standard form of z. (1)

(ii) If $z = 2x + (4 - y)\,i$ then find x and y. (1)

(iii) The modulus of z. (1)

(iv) In which Quadrant z lies ? (1)

37. Case Study 2 : During the math class, a teacher clears the concept of Permutation and combination to the 11th class students. After the class she asks the students some questions. One of the question was : how many numbers between 99 and 1000 can be formed such that

(i) Every digit is either 4 or 9. (1)

(ii) These is no restriction. (1)

(iii) Atleast one of digits is 9. (2)

38. Case Study 3 : Shelly (S) and Danju (D) are playing Ludo at home during Vacations. While rolling the dice, Shelly's sister noted the possible outcomes of the throw every time. Let A be the set of Players while B be the set of all possible outcomes.

Help her sister to find the following :

(i) $A \times B$ (1)

(ii) No. of Relations are possible, from A to B. (1)

(iii) If $C = \{2, 4\}$ then verify $A \times (B \cap C) = (A \times B) \cap (A \times C)$ (2)

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